Exponent Laws

The number 1 000 000 can be re-written as a repeated multiplication. Exponents are used as a short way to write the repeated multiplication.

$$1\,000\,000 = 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10^6$$

Standard Form   Repeated Multiplication   Exponential Form

In this example, the 10 is the base and the 6 is the exponent, the whole thing is called a power.

Investigation:

When you repeatedly fold a piece of paper in half, the number of layers increase with the number of folds. Fold a standard piece of paper, and copy and complete the table below.

Number of Folds	Number of Layers
1	2 or 2^1
2	2 x 2 = 4 or 2^2
3	$2 \times 2 \times 2 = 8$ or 2^3
4	$2 \times 2 \times 2 \times 2 = 16$ or 2^4
5	$2 \times 2 \times 2 \times 2 \times 2 = 32$ or 2^5
6	$2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6$

Hypothetically, if you could fold a piece of paper 10 times, how many layers would you have? 1024 (2^{10}) What about 12 times? 4096 (2^{12})

$3 \times 3 = 3^2$ is read as "three to the power of two" or more commonly as "three squared".

$2 \times 2 \times 2 = 2^3$ is read as "two to the power of three" or more commonly as "two cubed".

When an exponent is outside a pair of brackets, the exponent is applied to everything inside the brackets.

$\boxed{\text{ie: } (3y)^4 = (3y)(3y)(3y)(3y) = 3^4y^4 = 81y^4}$ Not $\cancel{3y^4}$

This is different from $3y^4 = 3(y)(y)(y)(y)$

State the base and the exponent of the following;

1. 5^3 b= 5 e= 3
2. 10^7
3. x^5 b= x e= 5
4. t^2

Write the following in exponential form;

5. $3 \times 3 \times 3 \times 3 \times 3$ $= 3^5$
6. $4 \times 4 \times 4 \times 4 \times 4 \times 4$ $= 4^6$
7. $10 \times 10 \times 10$ $= 10^3$
8. $6 \times 6 \times 6 \times 6$ $= 6^4$
9. $(m)(m)(m)(m)(m)(m)(m)$ $= m^7$

Write each number as a power of 10.

10. 100 $= 10^2$
11. 1000
12. 100 000
13. 10 000
14. 100 000 000
15. 10 000 000 10^7

Write as a power of 2.

16. 16 $= 2^4$
17. 64 $= 2$
18. 256 2^8
19. 2048 2^{11}
20. 512 2^9

Example # 1: Simplify $2^5 \times 2^3$

$2^5 = 2 \times 2 \times 2 \times 2 \times 2$
$2^3 = 2 \times 2 \times 2$
Therefore we have 2^8

To multiply powers with the same base, keep the base and add the exponents to simplify the expression:

$\times \quad +$

$\boxed{(x^m)(x^n) = x^{m+n}}$

Example # 2: Simplify $y^7 \div y^2$

$y^7 = (y)(y)(y)(y)(y)(y)(y)$
$y^2 = (y)(y)$
Therefore we have y^5

To divide powers with the same base, keep the base and subtract the exponents to simplify the expression:

$$x^m \div x^n = x^{m-n}$$

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Example # 3: Simplify the following;

a) $(2^4)^3$

$=(2^4)(2^4)(2^4)$
$=2^{12}$

b) $(y^2)^4$

$=(y^2)(y^2)(y^2)(y^2)$
$=y^8$

$$(x^m)^n \quad x^{mn}$$

To raise a power to another power, multiply the exponents.

$$\left(x^m\right)^n = x^{mn}$$

Simplify the following exponential expressions:

1. $5^3 \times 5^4 = 5^7$
2. $2^3 \times 2^7 = 2^{10}$
3. $10^6 \times 10 = 10^7$
4. $y^2 \times y^4 = y^6$
5. $a \times a^6 = a^7$
6. $4^5 \div 4^3 = 4^2$
7. $3^7 \div 3^6 = 3$
8. $(-5)^8 \div (-5) = (-5)^7$
9. $x^3 \div x^{-4} = x^7$
10. $(2^3)^8 = 2^{24}$
11. $(3^5)^2 = 3^{10}$
12. $(4^2)^7 = 4^{14}$
13. $(y^3)^3 = y^9$
14. $(m^2)^{-5} = m^{-10}$